MA 3513
Overview of topics for Midterm Examination 2

Timeline

	∼500 BCE	Pythagoreans study (among other things) Pell's equation and rational approximations to $\sqrt{2}$.
*	∼450 BCE	Zeno gives his paradox on the impossibility of motion.
*	early-300's BCE	Eudoxus does work with infinity, including the theory of proportions and the method of exhaustion.
*	∼300 BCE	Euclid publishes *Elements*, a highly influential and enduring textbook. Some topics include: that there are infinitely many primes, the Euclidean algorithm.
	200BCE - 200CE	Mathematicians in the Han dynasty in China develop the Euclidean algorithm and various consequences.
*	late 200's	Sun Zi states a special case of the Chinese Remainder Theorem.
	mid-200's	Diophantus studies rational solutions to polynomial equations. Method works from an obvious solution to a non-obvious one. (Chord and tangent method.)
*	499	Aryabhatta gives criterion for solution of linear Diophantine equation $ax + by = c$, gives algorithm for Chinese Remainder Theorem. Written in terse form (sutra).
*	628	Brahmagupta develops theory of composition of good triples for solving Pell-like equations.
	1247	Qin Jiushao characterized existence of inverses mod n; states and proves the general Chinese Remainder Theorem.
	mid-1600's	Fermat suggests "Fermat's Last Theorem" problem, develops Fermat primes, rediscovers cubic method of Diophantus.
	1670	Newton gives chord and tangent line interpretation to cubic method of Diophantus.
	1849	Euler shows even perfect numbers have the form $2^{n-1} \cdot (2^n - 1)$, where $2^n - 1$ is prime.
*	1872	Dedekind introduces Dedekind cuts, giving a rigorous definition of the real numbers.
	mid-1900's	R.L. Moore is active. Worked in topology. Known for the "Moore method" of teaching, where students are given definitions and theorems, and asked to fill in the proofs independently.
	1995	Wiles, joined by his student Taylor, prove Fermat's Last Theorem, that $x^n + y^n = z^n$ has solutions only when $n = 1$ or 2.
	2013	Zhang proves there are infinitely many prime gaps of at most 70 million. (Maynard and Polymath Project later reduce to gaps of around 250.)

Main ideas

1. Triangular, square, pentagonal, and hexagonal numbers.
2. Primes
   (a) There are infinitely many primes.
   (b) Fermat and Mersenne primes.
   (c) Perfect numbers.
   (d) Twin primes, Fermat's Last Theorem.
3. Diophantine chord and tangent method for cubics.
4. The Euclidean (pulverizer) Algorithm and consequences
   (a) Bézout's Lemma for $\mathbb{Z}$.
   (b) The prime divisor property; unique factorization into primes.
   (c) Connections with anthyphairesis and continued fractions.
   (d) Linear Diophantine equations $ax + by = c$.
5. Pell's equation and rational approximations of square roots
   (a) Good triples, composition of triples.
   (b) Self-composition of triples to find solutions to cases of Pell's equation.
6. Infinity in ancient Greece
   (a) Zeno's paradox.
   (b) Theory of proportions (Eudoxus), and its later development by Dedekind.
   (c) Method of exhaustion (Eudoxus).
7. Chinese Remainder Theorem
   (a) Modular arithmetic.
   (b) Motivation for modular problems from solar/lunar blended calendar.
   (c) Inverses mod n.