MA 3513
Overview of topics for Midterm Examination 1

Timeline

	∼1800 BCE	Babylonians write Pythagorean triples on clay tablets
∗	∼500 BCE	Pythagoras and his school prove the Pythagorean Theorem; discover $\sqrt{2}$ is irrational, so triangles may not be commensurable; maintain cult-like environment.
∗	mid-300's BCE	Menaechmus studies conic sections. Among other things, he shows how to use them to double the cube.
∗	∼300 BCE	Euclid publishes *Elements*, a highly influential and enduring textbook. Some topics include: the idea of proof based on axioms, a method for enumerating Pythagorean triples, polygonal numbers, ...
	∼150 BCE	Perseus studied toric sections (spiric sections).
	∼140 CE	Ptolemy studied epicycles, which he proposes as a model for planetary motion.
∗	mid-200's	Diophantus studies rational solutions to polynomial equations. Method works from an obvious solution to a non-obvious one.
	∼1000	Al-Kuji builds "3D compass" for conic sections.
	1609	Kepler observes planetary motion is elliptical. (Newton explains with gravity in 1687.)
∗	1637	Descartes introduces the Cartesian plane. Connects quadratic equations with conics. (Further developed by Wallis in 1655).
	1639	Descartes Angle Deficit Theorem for polyhedra
	1751	Euler's "$V - E + F = 2$" theorem for polyhedra
	1796	Gauss shows n-gons are constructible when primes dividing n are 2 and Fermat primes.
∗	1837	Wantzel examines ruler and compass constructions. Shows $\sqrt[3]{2}$ is not constructible, proves converse of Gauss' theorem on contructible n-gon.
	1872	Dedekind puts irrational numbers on a firm footing. (Gives rigorous definition.)
∗	1882	Lindemann shows $\pi, \sqrt{\pi}$ are transcendental, hence not constructible.
	1899	Hilbert uses Pythagorean theorem as the definition of distance in $\mathbb{R}^2$, completing the unification of numbers and geometry begun by Descartes.
	1900's	Erdős works. Introduces probabilistic method in combinatorics. Prolific, as evidenced by Erdős numbers. Has the influential idea of "Book" proofs. Later, Aigner and Ziegler turn into an actual book, *Proofs from the book*.

Main ideas

1. The Pythagorean Theorem
   (a) Result and proofs
   (b) Pythagorean triples, enumeration theorem of Euclid
   (c) Connection of Pythagorean triples with rational points on unit circle
   (d) Dedekind, Hilbert and distance in $\mathbb{R}^2$
2. Polyhedra
   (a) definition, the 5 regular polyhedra (theorem)
   (b) angle deficit; Descartes' Theorem
   (c) Euler's $V - E + F = 2$ Theorem
3. Ruler and compass constructions
   (a) Constructible lengths: meaning; $a + b$, $a - b$, $a \cdot b$, a/b, $\sqrt{a}$
   (b) Lemmas: you can construct equilateral triangles, right angles, parallel lines
   (c) Non-constructible lengths: $\sqrt[3]{2}$, π. (Doubling the cube, squaring the circle.)
   (d) Constructible n-gons
4. Conic sections; higher curves
   (a) Conic sections: idea, equations
   (b) Doubling the cube with conic sections
   (c) Toric sections idea
   (d) Epicycle idea
5. Polygonal numbers: definition, working with