

MA 4163 / 6163
Exam 2 – March 27, 2013

Name _____

General Instructions: Please answer the following, showing all your work and writing neatly. You may not refer to any books, notes, or calculators.
Three problems, 50 total points.

1. (5 points each) Quickies

- (a) Give the definition of a Sylow subgroup of a finite group G .
- (b) Find $|\text{Syl}_2(D_{10})|$ and $|\text{Syl}_5(D_{10})|$ for the dihedral group D_{10} .
- (c) Find a familiar group that is isomorphic to the Sylow 3-subgroup of \mathbb{Z}_{90} .
- (d) Let A and B be normal subgroups of G . Using the commutator subgroup G' , show that if G/A and G/B are both abelian, then $G/(A \cap B)$ is abelian.
- (e) Recall that a group G is *metabelian* if it has a normal subgroup A with both A and G/A abelian. Give an example of a metabelian group that is not abelian. (Find the normal subgroup A , and verify that your example works.)

2. (14 points) Show that there is no simple group of order 150.

Generous partial credit will be given for any restrictions you prove on the # of Sylow subgroups.

3. Suppose that G is a finite group with $N \triangleleft G$, and that $P \in \text{Syl}_p(G)$.

- (a) (2 points) Explain in 1-2 sentences why P acts on N by conjugation. (Why is the action well-defined?)
- (b) (9 points) If $|N| \not\equiv 1 \pmod{p}$, show that $C_G(P) \cap N \neq 1$.
Hint: let P act on N by conjugation, and consider the fixed point set.