

MA 4143 / 6143

Exam 1 – February 19, 2014

Name \_\_\_\_\_

**General Instructions:** Please answer the following, showing all your work and writing neatly. You may not refer to any books, notes, or calculators.  
50 total points.

1. (5 points each) Quickies. Remember to justify your answer.
  - (a) Give the definition of an isomorphism from a graph  $G$  to a graph  $H$ .
  - (b) Find the number of ways to put 9 elves and 4 goblins in a row, in such a way that no two goblins are next to one another.
  - (c) How many rearrangements are there of the word “MSSTATES”?
  - (d) How many subsets of  $\{1, \dots, 2n\}$  contain at least one element from  $\{1, \dots, n\}$ ?
  
2.
  - (a) (2 points) How many  $n \times n$  matrices are there with every entry either 0 or 1?
  
  - (b) (10 points) How many *upper triangular*  $n \times n$  matrices are there with every entry either 0 or 1? For full credit, simplify your answer as much as possible.  
(Recall from linear algebra that upper triangular means that the entry  $a_{i,j}$  is 0 if  $i > j$ . That is, the entries of the matrix are 0 below the diagonal; the entries on and above the diagonal can be either 0 or 1.)
  
3. (12 points) Find the number of graphs on  $n$  vertices that have no vertices of degree 0.  
Hint: Inclusion-exclusion may be helpful!
  
4. (6 points) If  $A$  is the adjacency matrix of the graph  $G$ , explain shortly why (when  $i \neq j$ ) the  $(i, j)$ th entry of the matrix  $A^2$  counts the number of paths of length 2 from  $v_i$  to  $v_j$ .