

Topics for Exam 2

1. The Correspondence Theorem and consequences
2. Commutators
 - (a) Using G' to detect abelian quotient groups
 - (b) (metabelian groups)
3. Group actions

These have been our main tool to prove serious theorems. Finding the right action to work with can be difficult, but you should be able to reproduce the proof of many of our theorems and/or similar results if told what action to work with.

 - (a) Basic definitions: Orbits, Transitivity, Stabilizers, Kernel
 - (b) “internal” group actions: conjugation, right multiplication, ...
 - (c) Using actions to find \triangleleft subgroups (via kernel).
 - (d) Using actions to find subgroups (via stabilizers).
 - (e) Orbit-Stabilizer Theorem and FCP
 - (f) fixed sets modulo p
4. Sylow subgroups
 - (a) Sylow Existence, Development, Conjugacy theorems
 - (b) Restrictions on $|\text{Syl}_p G|$:
 - i. $|\text{Syl}_p G| = [G : N_G(P)]$ by Sylow C $\implies |\text{Syl}_p G| \mid \left| \frac{G}{P} \right|$.
 - ii. $|\text{Syl}_p G| \equiv 1 \pmod{p}$.
 - iii. G/N injects into $\text{Sym}(\text{Syl}_p G)$. (Factorial conditions)
 - (c) Normal Sylow subgroups
5. Simple groups
 - (a) Definition, fact that \mathbb{Z}_p is simple.
 - (b) How to use Sylow subgroups to show no simple subgroups of a given order.
6. Finite p -groups P
 - (a) Have “big centers”: $Z(P) \neq 1$.