

MA 3053

Exam 1 – September 17, 2015

Name _____

General Instructions: Please answer the following, showing all your work and writing neatly. You may not refer to any books, notes, or calculators.
50 total points.

1. (4 points each) Quickies. Briefly justify your answer where appropriate.
 - (a) Give the definition of the Cartesian product $X \times Y$.
 - (b) Find a bijection $\lceil 6 \rceil \rightarrow \lceil 2 \rceil \times \lceil 3 \rceil$.
 - (c) If $f(x) = \sin x$, then find $\text{Image } f$.
 - (d) Give an example of a function $f : \mathbb{N} \rightarrow \mathbb{N}$ that is surjective but not injective.
 - (e) Let $X = \{x \in \mathbb{N} : x = p^a \text{ for some prime number } p \text{ and some } a \in \mathbb{N}\}$, and let $Y = \{x \in \mathbb{N} : x > 10\}$. Find a simple expression for $(X \cup Y)^c$.
 - (f) Give an example of a number from each of the sets $(\mathbb{R} \setminus \mathbb{Q}) \cap [0, 1]$ and $\mathbb{R} \setminus \left(\bigcup_{i=0}^{\infty} (i - \frac{1}{2}, i + \frac{1}{2}) \right)$, with brief explanations of why your numbers are in the desired sets.

2. (8 points) Recall that a binary (ternary) operation is an operation taking two (three) arguments.
 - (a) How many binary operations are there on $\lceil 2 \rceil$? How many of these are surjective?
 - (b) How many ternary operations are there on $\lceil 2 \rceil$? How many of these are surjective?

Remark: This has applications in logic design and computer engineering.

3. (10 points) Let X and Y be sets, with $A \subseteq X$ and $B \subseteq Y$. Let $f : X \rightarrow Y$ be a function.
 - (a) Show that $A \subseteq f^{-1}[f[A]]$.
 - (b) Show that if f is injective, then $A = f^{-1}[f[A]]$.
4. (8 points) If X and Y are any sets, possibly infinite, find a bijective function $X \times Y \rightarrow Y \times X$. (Of course you'll need to show that your function is actually a bijection!)