Scores	
1.	
2.	
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7.	
Total:	

MA 2733
Final Exam – December 11, 2015

Name _____ Section _____

7 T/F, several long answer. 68 points.

General Instructions: Please answer the following, without use of calculators. You may refer to up to 4 3x5 cards or a single letter-sized sheet of paper, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ The sequence a_n, recursively defined by $a_0 = 2$, $a_n = \frac{n}{4}a_{n-1}$ for $n \geq 1$, is a convergent sequence.

   (b) ____ The function $\sin(x^2 - \sqrt{2})$ has a Taylor series centered at $a = 0$.

   (c) ____ If $\lim_{n\to\infty} \frac{c_{n+1}}{c_n} = 3$, then the power series $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence $R = 3$.

   (d) ____ Any curve $y = f(x)$ can also be represented as a parametric equation.

   (e) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are orthogonal unit vectors, then $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is also a unit vector.

   (f) ____ The line $\vec{\mathbf{r}}(t) = \langle 3t - 1, t, 2t + 1\rangle$ is parallel to the plane $3x + y + 2z = 0$.

   (g) ____ If $\vec{\mathbf{v}}, \vec{\mathbf{u}}$, and $\vec{\mathbf{w}}$ are vectors in $\mathbb{R}^3$ such that $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are orthogonal, and $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are orthogonal, then $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}} + \vec{\mathbf{w}}$ are orthogonal.

2. Power series representations

(a) (5 points) Find a power series representation for $\dfrac{1}{x^2-4}$.

(b) (6 points) Find a power series representation for $\displaystyle\int e^{-x^2/2}\,dx$.

3. Convergence of power series. Consider the power series $\sum_{n=0}^{\infty} \frac{n}{4^n} \cdot x^n$.

   (a) (6 points) Find the radius of convergence of the given power series.

   (b) (6 points) Find the interval of convergence of the given power series.

4. (5 points) Find the vector equation of the tangent line to $\vec{\mathbf{r}}(t) = \left\langle t, \frac{t^2}{2}, \frac{t^3}{3} \right\rangle$ at $t = 1$.

5. Vector calculus. Consider the vector function $\vec{\mathbf{r}}(t) = \langle 2\cos t, 3\sin t, t\rangle$.

(a) (2 points) In 1-2 sentences, describe the curve traced by $\vec{\mathbf{r}}(t)$.

(b) (4 points) Calculate the unit tangent vector $\vec{\mathbf{T}}(t)$ for $\vec{\mathbf{r}}(t)$.

(c) (4 points) Calculate the curvature $\kappa(t)$ for $\vec{\mathbf{r}}(t)$.

(d) (3 points) Write $\vec{\mathbf{r}}''$ in terms of $\vec{\mathbf{T}}, \vec{\mathbf{N}}$ and $\vec{\mathbf{B}}$.

6. (6 points) The "explain" problem

   Explain why $\kappa(t) = \dfrac{\|\vec{\mathbf{r}}' \times \vec{\mathbf{r}}''\|}{\|\vec{\mathbf{r}}'\|^3}$, where κ is the curvature of the smooth vector function $\vec{\mathbf{r}}(t)$.

7. (a) (2 points) Draw a rough sketch of the polar curve $r = 2 - 2\sin\theta$.

   (b) (5 points) Find the area of the region that is both inside $r = 2 - 2\sin\theta$ and above the x-axis.