MA 2733

Examination 2 – October 20, 2015

Name ______________________________ Section __________

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ______________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ Since the series $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n^4+2}$ converges, the integral $\displaystyle\int_0^{\infty} \frac{1}{x^4+2}\,dx$ also converges (by the Integral Test).

   (b) ____ In the power series $\displaystyle\sum_{k=2}^{\infty} \frac{x^{2k}}{3k}$, the coefficient of x^2 is 1/3.

   (c) ____ If the power series $\displaystyle\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence 1, and $\displaystyle\lim_{n\to 0} c_n = 0$, then the interval of convergence is $[-1, 1]$.

   (d) ____ If the alternating series $\displaystyle\sum_{n=0}^{\infty} (-1)^n b_n$ conditionally converges, then $\displaystyle\sum_{n=0}^{\infty} |b_{2n}| = \infty$.

   (e) ____ The function $f(x) = \dfrac{1}{x}$ has a power series representation centered at $a = 0$.

2. Power series basics. Consider the power series $P(x) = \sum_{k=2}^{\infty} \frac{2^k}{(2k^2+1)} \cdot x^{2k+1}$.

(a) (2 points) Find the coefficients of x^0, x^3, x^6, and x^9 in $P(x)$.

(b) (4 points) Find the radius of convergence for $P(x)$.

(c) (6 points) Find the interval of convergence for $P(x)$.

3. Discuss convergence of the following series: determine whether each is absolutely convergent, conditionally convergent, or divergent.

(a) (6 points) $\sum_{n=1}^{\infty} \frac{\sin n + \cos n}{3n + n^2}$

(b) (5 points) $\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n} + \pi}$

(c) (5 points) $\sum_{n=3}^{\infty} \frac{(-1)^n}{n(\ln n)^3}$

4. Power series representations.

(a) (4 points) Find a power series representation for $f(x) = \dfrac{1}{4+x^4}$.

(To receive full credit, isolate the powers of x in your series.)

(b) (2 points) On what interval does your representation from part (a) converge? Why?

5. (6 points) The "explain" problem

Explain why $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 \cos n}{4^n}$ converges, without using the Ratio Test.