MA 2733
Examination 1 – September 25, 2013

Name ______________________________

5 T/F, 3 long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ______________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ If $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \bullet \vec{\mathbf{c}} = 0$, then $\vec{\mathbf{c}}$ lies in the same plane as $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.

   (b) ____ If $\vec{\mathbf{a}} \bullet \vec{\mathbf{b}} = 0$, then both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are unit vectors.

   (c) ____ The polar curve described by $\theta = 2$ (for θ on $[0, 2\pi]$) can also be described with a function $y = f(x)$.

   (d) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are unit vectors, then $\|\vec{\mathbf{v}} \times \vec{\mathbf{u}}\| = 1$.

   (e) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are unit vectors, then $-1 \leq \vec{\mathbf{v}} \bullet \vec{\mathbf{u}} \leq 1$.

2. Parametric and polar equations

(a) (4 points) Set up an integral for the arc length of the parametric curve given by $x = \sin^2 t$, $y = \cos t$ for t on the interval $[0, \pi/2]$. (You need not evaluate the integral.)

(b) (7 points) Find the area between the parametric curve $x = e^{-t}$, $y = e^{2t}$ and the x-axis for t between 0 and $\ln 2$.

(c) (7 points) By calculating an appropriate 2nd derivative, show that the cycloid described by $x = 5t - 5\sin t$, $y = 5 - 5\cos t$ is concave down except for at the points where t is a multiple of 2π.
At least 3 points will be given for correctly calculating an appropriate 1st derivative.

3. Vectors and straight-line geometry

(a) (3 points) In 1-2 sentences, explain what the phrase "dot products detect orthogonality" means.

(b) (5 points) Give the vector equation of a plane which is parallel to the plane $\langle 2, -1, 1\rangle \bullet (\vec{\mathbf{r}} - \langle 1, 1, 1\rangle)$ and which passes through the point $(0, 0, -1)$.

(c) (4 points) Write the plane with vector equation $\langle 2, -1, 1\rangle \bullet (\vec{\mathbf{r}} - \langle 1, 1, 1\rangle)$ in the form $\alpha x + \beta y + \gamma z = \lambda$.

(d) (4 points) Find a unit vector that is orthogonal to both $\langle 1, 1, 0\rangle$ and $\langle 2, 2, 2\rangle$.

4. (6 points) The "explain why"/proof problem
Explain why, if the parametric equation $x = f(t)$, $y = g(t)$ passes the vertical line test between $t = \alpha$ and $t = \beta$ (where $f(\alpha) < f(\beta)$ and $g(t) > 0$), then the area between the curve and the x-axis is

$$\int_{\alpha}^{\beta} g(t)f'(t)\,dt.$$