Scores	
1.	
2.	
3.	
4.	
5.	
Total:	50

MA 2733
Examination 3 – November 17, 2015

Name Harry Angstrom Section 1

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) T If $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are vectors in $\mathbb{R}^3$ with $\vec{\mathbf{a}} \cdot \vec{\mathbf{b}} = 0$, then $\vec{\mathbf{a}}$ is the normal vector to some plane containing $\vec{\mathbf{b}}$.

   (b) T The function $f(x) = \sqrt{1-x}$ has a Taylor series around $a = 0$.

   (c) T If $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are vectors in $\mathbb{R}^3$ so that $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ are orthogonal, and $\vec{\mathbf{a}}$ and $\vec{\mathbf{c}}$ are orthogonal, then necessarily $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}} + \vec{\mathbf{c}}$ are orthogonal.

   (d) T The vector function $\vec{\mathbf{r}}(t) = \langle \sin t, \sqrt{t^2+1}, |t| \rangle$ is continuous at t, for all values of t on $(-\infty, \infty)$.

   (e) T For any vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$ in $\mathbb{R}^3$, it holds that $\vec{\mathbf{a}} \times \vec{\mathbf{b}} = (\vec{\mathbf{a}} + \vec{\mathbf{b}}) \times \vec{\mathbf{b}}$.

2. Geometry of lines, planes, and curves

(a) (4 points) Find an equation of the form $ax + by + cz = d$ for the plane containing the points $(1, 0, 0)$, $(0, 2, 0)$, and $(0, 0, 1)$.

Parallel vectors

$$(0,2,0)-(1,0,0) = \langle -1,2,0\rangle$$
$$(0,0,1)-(1,0,0) = \langle -1,0,1\rangle$$
$$\times\text{ prod} = \langle 2, 1, 2\rangle$$

Equation

$$\vec{n}\circ\vec{r} = \vec{n}\circ\vec{r}_0$$

$$2x + y + 2z = 2\cdot1 + 1\cdot0 + 2\cdot0 = 2.$$

(b) (4 points) Determine whether the points lie all on a line, all on a plane, or neither: $(1, 1, 1)$, $(2, 2, 4)$, $(4, 2, 2)$, $(2, 1, 2)$.

Find plane containing 1st 3, as above:

Parallel vectors

$$(2,2,4)-(1,1,1) = \langle 1,1,3\rangle$$
$$(4,2,2)-(1,1,1) = \langle 3,1,1\rangle$$
$$\times\text{prod} = \langle -2, 8, -2\rangle$$

Equation $-2x + 8y - 2z = -2 + 8 - 2 = 4.$

Now plug in 4th pt:

$$-2\cdot2 + 8\cdot1 - 2\cdot2 = 0 \neq 4$$

So does not lie on same plane. Neither.

(c) (4 points) Find the vector equation of a line that passes through the point $(0, 0, 1)$ and that is orthogonal to the plane $2x - 2y + z = 3$.

Direction vector is normal to plane

ie, $\langle 2,-2,1\rangle$.

So $\vec{r}(t) = t\langle 2,-2,1\rangle + \langle 0,0,1\rangle.$

(d) (4 points) If $\vec{\mathbf{r}}(t) = \left\langle 2t+1, \frac{\cos t - 1}{t}, \frac{\sin^2 t}{t^2} \right\rangle$, then find $\lim_{t\to 0} \vec{\mathbf{r}}(t)$.

$$\lim_{t\to 0} \left\langle 2t+1,\ \frac{\cos t - 1}{t},\ \left(\frac{\sin t}{t}\right)^2 \right\rangle \quad \text{LH}$$

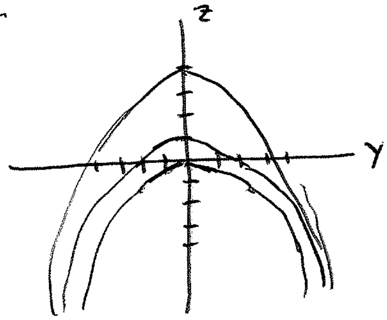
$$= \lim_{t\to 0} \left\langle 1,\ \frac{-\sin t}{1},\ 1 \right\rangle = \langle 1, 0, 1 \rangle.$$

3. (6 points) Consider the surface $z = x^2 - \frac{y^2}{4}$.

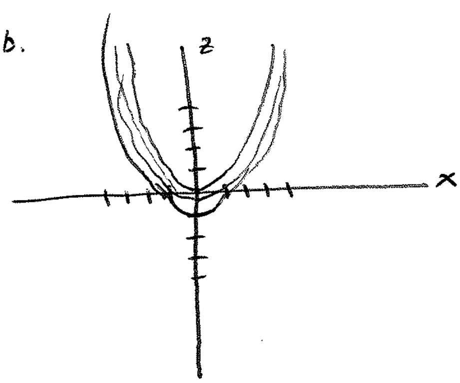
(a) Graph its traces in planes $x = 0$, $x = 1$, and $x = 2$ (all on the same axes).

(b) Graph its traces in planes $y = 0$, $y = 1$, and $y = 2$ (all on the same axes).

a.



b.



4. Power series representations

(a) (6 points) Find a power series representation for $x^4 \cos(\sqrt{2} \cdot x^2)$. To receive full credit, isolate the powers of x in your series.

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^{2k}$$

$$\Rightarrow x^4 \cos(\sqrt{2}\, x^2) = x^4 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} (\sqrt{2}\, x^2)^{2k}$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k 2^k}{(2k)!} x^{4k+4}$$

(b) (6 points) Find a power series representation for $\frac{3x}{x^2 - x - 2}$. To receive full credit, isolate the powers of x in your series.

Hint: partial fractions!

Partial fractions: $\frac{3x}{(x-2)(x+1)} = \frac{A}{(x-2)} + \frac{B}{x+1}$

$$3x = A(x+1) + B(x-2) \implies \begin{array}{l} A - 2B = 0 \\ A + B = 3 \end{array}$$

$$\implies A = 2, \quad B = 1$$

So above

$$= \frac{-2}{2-x} + \frac{1}{1+x} = \frac{-1}{1 - \frac{x}{2}} + \frac{1}{1-(-x)}$$

$$= \sum_{n=0}^{\infty} -\left(\frac{x}{2}\right)^n + \sum_{n=0}^{\infty} (-x)^n$$

$$= \sum_{n=0}^{\infty} \left(\frac{-1}{2^n} + (-1)^n\right) x^n$$

5. (6 points) The "explain" problem

Using Taylor's Coefficients Theorem directly, find the Taylor series around $a = 0$ for $f(x) = e^{-2x+1}$.

Take derivatives:

$$f(x) = e^{-2x+1}$$
$$f'(x) = -2e^{-2x+1}$$
$$f''(x) = (-2)^2 e^{-2x+1}$$
$$\vdots$$
$$f^{(n)}(x) = (-2)^n e^{-2x+1}$$
$$\vdots$$

Plug in 0:

$$f(0) = e^{0+1} = e$$
$$f'(x) = -2e$$
$$f''(x) = (-2)^2 e$$
$$\vdots$$
$$f^{(n)}(0) = (-2)^n \cdot e$$

Apply TCT:

$$c_n = \frac{f^{(n)}(0)}{n!} = \frac{(-2)^n \cdot e}{n!}$$

So

$$e^{-2x+1} = \sum_{n=0}^{\infty} \frac{(-2)^n \cdot e}{n!} x^n$$