MA 2733
Examination 1 – September 22, 2015

Name Harry Angstrom

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature

0. (1 point) My best estimate of my # of absences from MA 2733 lecture is ____. (Any good-faith estimate will be marked correct. Honor code applies.)

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) T If $\sum_{n=2}^{\infty} a_n = 2$, then necessarily $\lim_{n\to\infty} a_n = 0$.

   (b) F If $\lim_{n\to\infty} a_n = 2$, then necessarily $\sum_{n=2}^{\infty} a_n$ converges.

   (c) F If $\sum_{n=1}^{\infty} a_n = \frac{1}{2}$, then $a_n = \frac{1}{3^n}$.

   (d) T The sequence $a_n = \frac{1}{n!}$ can be expressed recursively as $a_0 = 1$, $a_n = \frac{a_{n-1}}{n}$ for $n \geq 1$.

   (e) F The parametric curve given by $x = t^2$, $y = t$ for t on $(-\infty, \infty)$ is the same as that given by the equation $y = \sqrt{x}$.

2. Parametric and polar equations

(a) (4 points) Find the slope of the tangent line to the parametric curve $x = 2\cos t$, $y = 4\sin t$ at $t = \pi/3$.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4\cos t}{-2\sin t} = \frac{-2\cos t}{\sin t}$$

At $t = \pi/3$:

$$\text{slope } m = \frac{-2\cos \pi/3}{\sin \pi/3} = \frac{-2\cdot\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{-2}{\sqrt{3}}$$

(b) (5 points) Find the area inside the ellipse bounded by the parametric curve $x = 2\cos t$, $y = 4\sin t$ for t on $[0, 2\pi]$.

By area formula for param + symmetry:

$$A = 2\cdot\int_{\pi}^{0} 4\sin t \cdot \frac{d}{dt}[2\cos t]\,dt = 2\cdot\int_{\pi}^{0} -8\sin^2 t\,dt = \int_0^{\pi} 16\sin^2 t\,dt$$

Use trig id:

$$= \int_0^{\pi} 8 - 8\cos 2t\,dt = \left[8t - 4\sin 2t\right]_0^{\pi} = 8\pi.$$

(c) (6 points) Find the length of the polar curve $r = \cos^2(\theta/2)$ where $0 \le \theta \le 2\pi$.

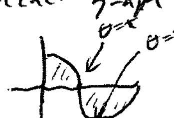
Then $r' = 2\cdot\cos\frac{\theta}{2}\cdot\left(-\sin\frac{\theta}{2}\right)\cdot\frac{1}{2} = -\cos\frac{\theta}{2}\sin\frac{\theta}{2}$. (By 2 applications of Chain rule.)

So Length formula for polar gives

$$\int_0^{2\pi}\sqrt{(r')^2 + r^2}\,d\theta = \int_0^{2\pi}\sqrt{\cos^2\frac{\theta}{2}\cdot\sin^2\frac{\theta}{2} + \cos^4\frac{\theta}{2}}\,d\theta$$

$$= \int_0^{2\pi}\sqrt{\cos^2\frac{\theta}{2}\underbrace{\left(\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2}\right)}_{\text{Pythagorean Thm.}}}\,d\theta = \int_0^{2\pi}\sqrt{\cos^2\frac{\theta}{2}\cdot 1}\,d\theta = \int_0^{2\pi}\left|\cos\frac{\theta}{2}\right|d\theta$$

Recall graph of $\cos\frac{\theta}{2}$.



So above is (by symmetry)

$$= 2\cdot\int_0^{\pi}\cos\frac{\theta}{2}\,d\theta = 2\cdot\left[2\sin\frac{\theta}{2}\right]_0^{\pi} = 4.$$

3. (6 points) The "explain" problem.

Explain why $\sum_{n=0}^{\infty} r^n = \infty$ when $r \geq 1$.

When $r=1$, $\lim_{n\to\infty} r^n = \lim_{n\to\infty} 1^n = \lim_{n\to\infty} 1 = 1$,

while when $r>1$, $\lim_{n\to\infty} r^n = \infty$.

So by nth term test, series diverges.

Since terms are positive, diverges to ∞.

4. (6 points each) For each of the following series, determine whether it is convergent or divergent. If it is convergent, find its sum.

(a) $\sum_{n=3}^{\infty} 2 \cdot \cos^n(3)$.

Geometric series w/

$a = 2 \cdot \cos^3(3)$

$r = \cos 3$, which is on $(-1,1)$ as 3 is not a multiple of π,

So <u>converges</u> to $\dfrac{2 \cdot \cos^3(3)}{1-\cos(3)}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{\sqrt{n^2+n}}$.

Simplify to $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{\sqrt{n(n+1)}} - \frac{\sqrt{n}}{\sqrt{n(n+1)}} = \sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}}\right)$

which telescopes, w/

$$S_N = \left(1 - \cancel{\tfrac{1}{\sqrt{2}}}\right) + \left(\cancel{\tfrac{1}{\sqrt{2}}} - \cancel{\tfrac{1}{\sqrt{3}}}\right) + \cdots + \left(\cancel{\tfrac{1}{\sqrt{N}}} - \tfrac{1}{\sqrt{N+1}}\right) = 1 - \frac{1}{\sqrt{N+1}}.$$

as $N \to \infty$, $S_N \to 1$,

so series converges to 1.

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{n3^n}{3^n+2^n}$.

Apply nth term test:

$$\lim_{n\to\infty} \frac{n\cdot 3^n}{3^n+2^n} \cdot \frac{3^{-n}}{3^{-n}} = \lim_{n\to\infty} \frac{n}{1+\left(\frac{2}{3}\right)^n}$$

∞ $\qquad$ 0

$= \infty$

and as limit of nth term is not 0,

series diverges.