MA 2733

Examination 3 – November 20, 2013

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5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

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1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) T A series which converges conditionally also converges.

(part of definition)

(b) T If $\sum_{n=0}^{\infty} a_n$ converges, then $\lim_{n\to\infty} |a_n| = 0$.

$\lim_{n\to\infty} a_n = 0$, so same for absolute value,

(c) F If $\sum_{n=0}^{\infty} f(n)$ converges, then $\int_0^{\infty} f(x)\,dx$ converges.

Conditions cts, +, decreasing are required

(d) F In $f(x) = \sum_{k=0}^{\infty} x^{2k}$ (considered as a power series), the coefficient of x^3 does not exist.

It is 0.

(e) F In the power series $f(x) = \sum_{k=0}^{\infty} k \cdot x^{2k}$, the coefficient of x^5 is 5.

It is 0.

2. Discuss convergence of the following series: determine whether each is absolutely convergent, conditionally convergent, or divergent.

(a) (6 points) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{n-\pi}$

Since $0 < \frac{1}{n} < \frac{1}{n-\pi}$ for $n \geq 4$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n} = \infty$,

the series diverges by the DCT.

(b) (6 points) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+3n-5}{n!}$

Apply Ratio Test:

$$L = \lim_{n\to\infty} \left| \frac{\frac{(n+1)^2+3(n+1)-5}{(n+1)!}}{\frac{n^2+3n-5}{n!}} \right| = \lim_{n\to\infty} \frac{n!}{(n+1)!} \cdot \frac{n^2+5n-1}{n^2+3n-5}$$

$$= \lim_{n\to\infty} \frac{1}{n+1} \frac{1+\frac{5}{n}-\frac{1}{n^2}}{1+\frac{3}{n}-\frac{5}{n^2}} = \lim_{n\to\infty} \frac{1}{n+1} = 0 < 1$$

So series converges (absolutely).

(c) (6 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{n^2+\pi}$.

Since $\left| \frac{(-1)^n}{n^2+\pi} \right| = \frac{1}{n^2+\pi} < \frac{1}{n^2}$

and $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges,

the series converges absolutely.

(d) (8 points) $\sum_{n=0}^{\infty} \frac{(-1)^n}{n+\pi}$.

First: $n+\pi < 5n$ for $n \geq 1$

So $\left|\frac{(-1)^n}{n+\pi}\right| = \frac{1}{n+\pi} > \frac{1}{5n}$ for $n \geq 1$

and by DCT the series does not absolutely converge.

Then: apply AST.

$\frac{1}{n+\pi} > 0$ (alternating)

$\frac{1}{n+\pi} > \frac{1}{n+1+\pi}$ (decreasing)

and $\lim_{n\to\infty} \frac{1}{n+\pi} = 0$

So the series converges conditionally.

3. (6 points) Discuss the convergence of $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

Hint: the Integral Test may be helpful.

Since $\frac{1}{x \ln x}$ is cts, positive and decreasing on the interval $[2, \infty)$

the Integral Test tells us

$\sum_{n=2}^{\infty} \frac{1}{n \ln 2}$ and $\int_2^{\infty} \frac{1}{x \ln x}\,dx$ are equiconvergent.

Calculate:

$$\int_2^{\infty} \frac{1}{x \ln x} = \int_{\ln 2}^{\infty} \frac{1}{u}\,du = \left[\ln u\right]_{\ln 2}^{\infty} = \lim_{t\to\infty} \ln t - \ln(\ln 2)$$

substitute $u = \ln x$

$du = \frac{1}{x}\,dx$

$= \infty$

and since the integral diverges to ∞, the series also diverges.

4. (8 points) The "explain" problem.

(a) Explain how to rearrange the terms of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ to obtain a series converging to 0.

First take enough + terms to be >0
(possible since $\sum(+\text{terms}) = \infty$)

Then enough − terms to be <0

Then enough + — — — >0 again

— — — — — — — <0 — —

Repeat forever. Each step can be carried out
since $\sum(+\text{terms}) = \infty$
and $\sum(-\text{terms}) = -\infty$

(b) Explain why the same argument cannot be applied to $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$. (Where does your argument from (a) fail with the latter series?)

The positive terms of $\sum \frac{(-1)^n}{n^2}$ do not add to ∞
and the − terms — — do not add to $-\infty$.

As a result, we will "run out of" terms at
some step of the argument in (a), w/o changing the sign.