MA 2733
Examination 2 – October 23, 2013

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5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ♡

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) F The curvature of the line $\vec{\mathbf{r}}(t) = \langle t, t, t\rangle$ is not defined.

   (b) T For the vector function $\vec{\mathbf{r}}(t) = \langle t, t^2, 2t^3\rangle$, the unit normal vector $\vec{\mathbf{N}}$ is orthogonal to $\vec{\mathbf{N}}'$.

   (c) F If a_n is an increasing sequence, then $\lim\limits_{n\to\infty} a_n$ converges.

   (d) T Suppose f is a continuous function with $\lim\limits_{x\to\infty} f(x) = 0$. If $a_n = f(n)$, then $\lim\limits_{n\to\infty} a_n = 0$.

   (e) T If $\vec{\mathbf{r}}(t)$ is a vector function with unit normal vector $\vec{\mathbf{T}}(t)$, then

$$\left(\vec{\mathbf{T}} + \vec{\mathbf{T}}'\right) \cdot \left(\vec{\mathbf{T}} + \vec{\mathbf{T}}'\right) = \left\|\vec{\mathbf{T}}\right\|^2 + \left\|\vec{\mathbf{T}}'\right\|^2.$$

2. (11 points) Write $\vec{\mathbf{r}}''(t)$ in the $\vec{\mathbf{T}}, \vec{\mathbf{N}}, \vec{\mathbf{B}}$ frame for the vector function $\vec{\mathbf{r}}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$. (You need not calculate $\vec{\mathbf{T}}$, $\vec{\mathbf{N}}$, or $\vec{\mathbf{B}}$.)

$$\vec{r}'' = v'\vec{T} + v^2 K \vec{N}$$

where $v = \|\vec{r}'\|$ and $K = \frac{\|\vec{r}' \times \vec{r}''\|}{\|\vec{r}'\|^3} = \frac{\|\vec{r}' \times \vec{r}''\|}{v^3}$ is the curvature.

Then $\vec{r}' = \langle -4\sin t, 4\cos t, 3\rangle$

$$\Rightarrow v = \|\vec{r}'\| = \sqrt{4^2\sin^2 t + 4^2\cos^2 t + 3^2}$$
$$= \sqrt{16+9} = 5$$

$$\Rightarrow v' = 0$$

and $\vec{r}'' = \langle -4\cos t, -4\sin t, 0\rangle$

$$\Rightarrow K = \frac{\|\vec{r}' \times \vec{r}''\|}{v^3} = \frac{\|\langle +12\sin t, 12\cos t, \cancel{16\sin^2 t + 16\cos^2 t}\ 16\rangle\|}{v^3}$$

$$= \frac{\sqrt{12^2\sin^2 t + 12^2\cos^2 t + 16}}{v^3} = \frac{\sqrt{144+256}}{v^3} = \frac{20}{v^3}$$

So $\vec{r}'' = 0\vec{T} + 5^2 \cdot \frac{20}{5^3}\vec{N} + 0\vec{B} = \boxed{4\vec{N}.}$

3. (9 points) Find $\lim_{n\to\infty} \frac{2^{\sin(\frac{\pi}{2}n)}}{n^2}$. (Make sure you explain how you found your answer!)

Since $-1 \leq \sin(\frac{\pi}{2}n) \leq 1$ for all n,

we have $2^{-1} \leq 2^{\sin(\frac{\pi}{2}n)} \leq 2^1$ for all n

So $\frac{1}{2n^2} \leq \frac{2^{\sin(\frac{\pi}{2}n)}}{n^2} \leq \frac{2}{n^2}$.

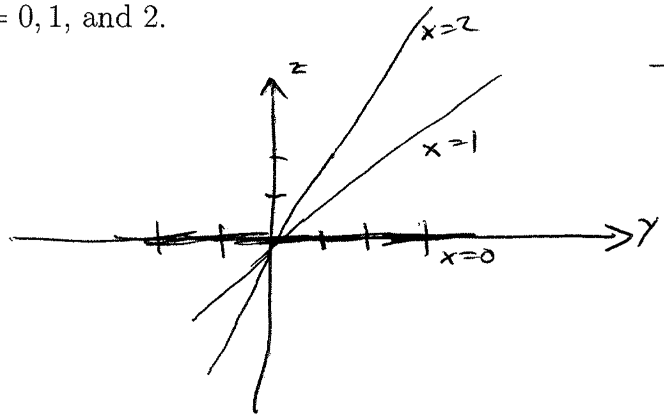
As both $\frac{1}{2n^2}$ and $\frac{2}{n^2}$ go to 0 as $n \to \infty$,

the sequence is sandwiched between them,

and $\boxed{\lim_{n\to\infty} \frac{2^{\sin(\frac{\pi}{2}n)}}{n^2} = 0 \text{ by Sandwich / Squeeze Thm.}}$

4. Planes and surfaces

(a) (7 points) On the same axis, draw the traces of the surface $z = xy$ in the planes $x = 0, 1,$ and 2.



x	
0	$z=0$
1	$z=y$
2	$z=2y$

(b) (2 points) Consider the line parallel to the vector $\langle 2, 1, 2\rangle$ and passing through $(1, 0, 0)$. Find a point other than $(1, 0, 0)$ on this line.

e.g. $\boxed{(3,1,2)} = (2,1,2) + (1,0,0)$.

(c) (3 points) Find 3 distinct points on the plane $x - y - 2z = 1$.

Eg $(1,0,0)$
$(0,-1,0)$
$(0,0,-\frac{1}{2})$.

(Found by setting 2 coordinates to be 0 and solving for the 3rd.)

5. The "explain" problem.

(a) (4 points) State and verify the "sum rule" for $\frac{d}{dt}[\vec{\mathbf{v}}(t) + \vec{\mathbf{u}}(t)]$.

$$\frac{d}{dt}[\vec{v}(t) + \vec{u}(t)] = \frac{d}{dt}[\vec{v}(t)] + \frac{d}{dt}[\vec{u}(t)]$$

Since

$$\frac{d}{dt}[\vec{v}(t) + \vec{u}(t)] = \lim_{h \to 0} \frac{(\vec{v}(t+h) + \vec{u}(t+h)) - (\vec{v}(t) + \vec{u}(t))}{h}$$

$$= \lim_{h \to 0} \frac{\vec{v}(t+h) - \vec{v}(t)}{h} + \frac{\vec{u}(t+h) - \vec{u}(t)}{h}$$

$$= \frac{d}{dt}[\vec{v}(t)] + \frac{d}{dt}[\vec{u}(t)].$$

(b) (4 points) State and verify the "dot product rule" for $\frac{d}{dt}[\vec{\mathbf{v}}(t) \cdot \vec{\mathbf{u}}(t)]$.

$$\frac{d}{dt}[\vec{v}(t) \cdot \vec{u}(t)] = \frac{d}{dt}[v_1(t)u_1(t) + v_2(t)u_2(t) + \dots]$$

$$= [v_1'(t)u_1(t) + v_1(t)u_1'(t) + \dots]$$

$$= \vec{v}'(t) \cdot \vec{u}(t) + \vec{v}(t) \cdot \vec{u}'(t).$$