Scores	
1.	
2.	
3.	
4.	
5.	
Total:	

MA 2733
Examination 3 – November 17, 2016

Name Harry Angstrom Section 8

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ʊ3

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) F If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are vectors so that the angle between $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ is $\pi/3$, then $\mathrm{proj}_{\vec{\mathbf{u}}}\vec{\mathbf{v}} = \vec{\mathbf{u}}/2$. (direction is wrong)

   (b) F If $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ are vectors so that $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are orthogonal, then $\vec{\mathbf{u}} \times \vec{\mathbf{v}} = \vec{\mathbf{w}}$.

   (c) F If $\vec{\mathbf{u}}, \vec{\mathbf{v}}, \vec{\mathbf{w}}$ are vectors so that $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are orthogonal, then $(\vec{\mathbf{u}} \bullet \vec{\mathbf{v}}) \times \vec{\mathbf{w}} = 0$. scalar cross a vector??

   (d) T If vectors $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are both parallel to a plane, then $\vec{\mathbf{u}} + 2\vec{\mathbf{v}}$ is parallel to the same plane.

   (e) T If vector $\vec{\mathbf{u}}$ is parallel to the xy-plane, then $\vec{\mathbf{u}} \bullet \vec{\mathbf{k}} = 0$.

2. (6 points) Taylor series and power series representations

Find a power series representation of $\dfrac{e^{x^2}-1}{x}$ (for $x \neq 0$) centered at $a = 0$. To receive full credit, isolate the powers of x in your series.

$$e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k \qquad \text{(From class)}$$

So $$e^{x^2} = \sum_{k=0}^{\infty} \frac{1}{k!} (x^2)^k = \sum_{k=0}^{\infty} \frac{1}{k!} x^{2k}$$

So $$e^{x^2} - 1 = \sum_{k=1}^{\infty} \frac{1}{k!} x^{2k}.$$ (Note indices of summation)

So $$\frac{e^{x^2}-1}{x} = x^{-1} \cdot (e^{x^2}-1) = \sum_{k=1}^{\infty} \frac{1}{k!} x^{2k-1}.$$

3. Vector functions and curves

For the vector function $\vec{\mathbf{r}}(t) = \left\langle 2t, t^2, \dfrac{t^3}{3} \right\rangle$:

(a) (5 points) Find the tangent vector $\vec{\mathbf{r}}'(t)$ and unit tangent vector $\vec{\mathbf{T}}(t)$.

$$\vec{r}\,'(t) = \langle 2, 2t, t^2 \rangle$$

$$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \frac{\langle 2, 2t, t^2 \rangle}{\sqrt{4+4t^2+t^4}}$$

(b) (6 points) Find the length of the curve traced as $0 \le t \le 2$.

$$L = \int_0^2 \|\vec{r}\,'(t)\|\,dt = \int_0^2 \sqrt{4+4t^2+t^4}\,dt$$
$$= \int_0^2 \sqrt{(2+t^2)^2}\,dt$$
$$= \int_0^2 2+t^2\,dt$$
$$= \left[2t + \frac{t^3}{3}\right]_0^2 = 4 + \frac{8}{3}.$$

4. Lines, planes, and spheres

(a) (3 points) Explain briefly why the point $(2\sqrt{2}, -1, 4)$ lies on the sphere of radius 5 centered at the origin.

Since the distance from the origin to the point is $\sqrt{(2\sqrt{2})^2 + (-1)^2 + 4^2}$
$= \sqrt{8+1+16} = \sqrt{25} = 5$

(b) (5 points) Find a vector equation of the line passing through the origin and $(2\sqrt{2}, -1, 4)$.

$$\vec{r}(t) = \langle 0,0,0\rangle + t\langle 2\sqrt{2}, -1, 4\rangle$$
$$= t\langle 2\sqrt{2}, -1, 4\rangle.$$

(c) (4 points) Find an equation of the plane that contains the point $(2\sqrt{2}, -1, 4)$ and is orthogonal to the line from part (b).

Normal vector $\langle 2\sqrt{2}, -1, 4 \rangle$ from part (b),

$$\langle 2\sqrt{2}, -1, 4\rangle \cdot \left(\langle x, y, z\rangle - \langle 2\sqrt{2}, -1, 4\rangle \right) = 0$$

$$2\sqrt{2}x - y + 4z = 25.$$

(d) (5 points) Find an equation of the plane containing the origin and the points $(2\sqrt{2}, -1, 4)$ and $(2\sqrt{2}, 1, 4)$.

2 parallel vectors

$$\vec{i} \quad \vec{j} \quad \vec{k}$$

$$(2\sqrt{2}, -1, 4) - (0,0,0) = \langle 2\sqrt{2}, -1, 4\rangle$$

$$(2\sqrt{2}, 1, 4) - (0,0,0) = \langle 2\sqrt{2}, 1, 4\rangle$$

$$\text{x-prod} = \langle -8, 0, 4\sqrt{2}\rangle$$

Equation $\langle -8, 0, 4\sqrt{2}\rangle \cdot \langle x, y, z\rangle = 0$

$$-8x + 4\sqrt{2}z = 0$$

5. (6 points) The "explain" problem

Explain why, if $f(x) = \sum_{n=0}^{\infty} c_n x^n$, then $f^{(n)}(0) = n! \cdot c_n$.

Take k derivatives of $f(x)$:

$$f'(x) = \sum_{n=0}^{\infty} c_{n+1} (n+1) x^n$$

$$f''(x) = \sum_{n=0}^{\infty} c_{n+2} (n+2)(n+1) x^n$$

$$\vdots$$

$$f^{(k)}(x) = \sum_{n=0}^{\infty} c_{n+k} (n+k) \cdot (n+k-1) \cdots (n+1) x^n$$

Plugging in $x=0$ gives 0 in all terms but constant

Eg $f(0) = c_0 + c_1 \cdot 0 + c_2 \cdot 0^2 + \cdots$

$= c_0$

So $f^{(k)}(0) = k! \cdot c_{0+k} = k! \cdot c_k$, as desired.