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MA 2733

Examination 2 – October 20, 2016

Name Harry Angstrom Section 8

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) T If the power series $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = -4$, then it must necessarily converge at $x = 4/3$. Radius of convergence at least 4.

(b) T If the power series $\sum_{n=0}^{\infty} c_n x^n$ has radius of convergence 1, and $0 \le c_n \le n^{-2}$ for all n, then the interval of convergence is $[-1, 1]$.

(c) T If $\lim_{n\to\infty} \frac{c_{n+1}}{c_n} = \frac{\pi}{4}$, then the series $\sum_{n=1}^{\infty} c_n$ converges. Ratio Test

(d) F We can consider $\sum_{n=0}^{\infty} \frac{x^{n-1}}{3^n}$ to be a power series. $\frac{1}{x}$ term

(e) T We can consider $x^3 - x + 1$ to be a power series. Polynomials are power series w/ lots of zeroes.

2. Power series basics. Consider the power series $P(x) = \sum_{n=1}^{\infty} \frac{9n}{2^n \cdot \sqrt{n+1}} \cdot x^n$

(a) (2 points) Find the coefficients of x^0, x^1, and x^2 in $P(x)$.

Coef of: $\underline{x^0}$: 0 (Series starts at $n=1$)

x^1: $9/2\sqrt{2}$

x^2: $18/4\sqrt{3}$.

(b) (5 points) Find the radius of convergence for $P(x)$.

Ratio Test:

$$\lim_{n\to\infty} \left| \frac{\frac{9(n+1)\, x^{n+1}}{2^{n+1}\sqrt{n+2}}}{\frac{9n\, x^n}{2^n \sqrt{n+1}}} \right| = \lim_{n\to\infty} \frac{\cancel{9}(n+1)}{\cancel{9}n} \cdot \frac{2^n}{2^{n+1}} \cdot \sqrt{\frac{n+1}{n+2}} \cdot \frac{|x|^{n+1}}{|x|^n}$$

$$= \lim_{n\to\infty} \left(1+\tfrac{1}{n}\right) \cdot \frac{1}{2} \cdot \sqrt{\frac{1+1/n}{1+2/n}} \cdot |x|$$

$$= (1+0) \cdot \frac{1}{2} \cdot \sqrt{\frac{1+0}{1+0}} \cdot |x| = \frac{|x|}{2}.$$

Series converges when $\frac{|x|}{2} < 1$

ie, when $|x| < 2$.

$\boxed{R = 2}$

(c) (5 points) Find the interval of convergence for $P(x)$.

Check endpts!

$\underline{x=2}$: $\sum_{n=1}^{\infty} \frac{9n}{\sqrt{n+1}}$, and as $\lim_{n\to\infty} \frac{9n}{\sqrt{n+1}} = \infty$,

diverges by nth Term Test,

$\underline{x=-2}$: As $\lim_{n\to\infty} \frac{9n}{\sqrt{n+1}} = \infty$,

$\lim_{n\to\infty} (-1)^n \frac{9n}{\sqrt{n+1}}$ DNE (oscillates)

so diverges by nth Term Test,

$\boxed{\text{Interval of convergence } (-2,2).}$

3. Find a power series representation centered at 0 for the following functions. For full credit, isolate the power of x in your series.

(a) (5 points) $\dfrac{1}{1+2x^2} = \dfrac{1}{1-(-2x^2)} = \sum_{k=0}^{\infty} (-2x^2)^k = \sum_{k=0}^{\infty} (-2)^k x^{2k}$.

(b) (6 points) $\ln(2-x) = \int \frac{-1}{2-x}\,dx = \int \frac{-1}{2} \cdot \frac{1}{1-\frac{x}{2}}\,dx$

$$= \int \sum_{n=0}^{\infty} -\frac{1}{2} \cdot \left(\frac{x}{2}\right)^n dx$$

$$= \int \sum_{n=0}^{\infty} \frac{-1}{2^{n+1}} x^n\,dx$$

$= C + \sum_{n=1}^{\infty} \frac{-1}{2}$ [redacted] $n \cdot \frac{x^n}{n}$

Solve for C:

$$\ln(2-0) = \ln 2 = C + \sum 0 = C$$

So

$$\ln(2-x) = \ln 2 + \sum_{n=1}^{\infty} \frac{-1}{2^n} \frac{x^n}{n}$$

4. (6 points) The "explain" problem.

Explain why $\dfrac{d}{dx}\left[\sum_{n=0}^{\infty} c_n x^n\right] = \sum_{n=0}^{\infty} c_{n+1} \cdot (n+1) \cdot x^n$.

$$\frac{d}{dx}\left[\sum_{n=0}^{\infty} c_n x^n\right] = \sum_{n=0}^{\infty} \frac{d}{dx}[c_n x^n]$$

$= \sum_{n=1}^{\infty} c_n \cdot n \cdot x^{n-1}$ (Note [redacted] starting value of n.)

$= \sum_{n=0}^{\infty} c_{n+1} (n+1) x^n$ (by reindexing).

5. Discuss convergence of the following series: determine whether each is absolutely convergent, conditionally convergent, or divergent.

(a) (6 points) $\sum_{n=0}^{\infty} \frac{\cos 2n}{2^n}$

$$\left|\frac{\cos 2n}{2^n}\right| = \frac{|\cos 2n|}{2^n} \leq \frac{1}{2^n}$$

Since $\sum_{n=0}^{\infty} \frac{1}{2^n}$ converges, $\sum_{n=0}^{\infty} \frac{\cos 2n}{2^n}$ also converges by DCT,

(b) (5 points) $\sum_{n=0}^{\infty} \frac{(-4)^n}{n!}$

Apply Ratio Test:

$$\lim_{n\to\infty} \left| \frac{\frac{(-4)^{n+1}}{(n+1)!}}{\frac{(-4)^n}{n!}} \right| = \lim_{n\to\infty} \frac{n!}{(n+1)!} \cdot \frac{4^{n+1}}{4^n}$$

$$= \lim_{n\to\infty} \frac{1}{(n+1)} \cdot 4 = 0$$

as $0<1$, series converges.