Scores	
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Total:	

MA 2733
Examination 1 – September 22, 2016

Name Harry Angstrom Section 8

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature

0. (1 point) My best estimate of my # of absences from MA 2733 lecture is 0. (Any good-faith estimate will be marked correct. Honor code applies.)

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) T The sequence $a_n = \sin(\pi n)$ converges.

   (b) F If $y = \sin t$ and $x = \cos t + t^2$ for t on $[0, \pi]$, then $\dfrac{d^2y}{d^2x} = \dfrac{-\sin t}{2 - \cos t}$ on the same interval.

   (c) T Every polar curve $r = \rho(t)$ can also be written as a parametric curve in the form $y = g(t), x = f(t)$.

   (d) F Suppose $y = g(t)$ and $x = f(t)$ for t on $[0, \infty)$ is a parametric curve. If $g'(1) = f'(1) = 0$, then the curve necessarily has a horizontal tangent at $t = 1$.

   (e) T If a parametric curve $y = g(t), x = f(t)$ passes the vertical line test, then it can always be written as a function $y = h(x)$.

2. (5 points) Find the limit of the sequence $a_n = \dfrac{n \cdot 2^n}{3^n}$.

$$\lim_{n\to\infty} \frac{n \cdot 2^n}{3^n} = \lim_{n\to\infty} \frac{n}{(3/2)^n} \overset{\text{l'H}}{=} \lim_{n\to\infty} \frac{1}{\ln\frac{3}{2} \cdot (3/2)^n}$$

$$= 0.$$

3. Areas

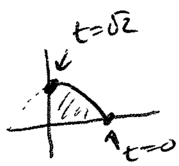
(a) (6 points) Find the area bounded by the curve $r = 2\cos\theta$ and in the first quadrant.

$$A = \int_0^{\pi/2} \frac{1}{2}(2\cos\theta)^2 \, d\theta = \int_0^{\pi/2} 2\cos^2\theta \, d\theta$$

$$= \int_0^{\pi/2} 1 + \cos 2\theta \, d\theta = \left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$= \left(\frac{\pi}{2} + 0\right) - (0 + 0)$$

$$= \frac{\pi}{2}$$



(b) (6 points) Find the area bounded by the curve $x = 2 - t^2$, $y = \sin t^2$ and in the first quadrant. (You may leave your answer in terms of $\sin 2$ and/or $\cos 2$.)

$x \geq 0$ and $y \geq 0$,

Have $x \geq 0$ when $|t| \leq \sqrt{2}$. (ie, when $t^2 \leq 2$)

Have $y \geq 0$ when t^2 on $[0, \pi/2]$, ie, when $t > 0$.

Intersected between $t = \sqrt{2}$ and $t = 0$:

$$A = \int_{\sqrt{2}}^{0} y \, dx = \int_{\sqrt{2}}^{0} \sin t^2 \cdot (-2t) \, dt$$

subst $u = t^2$

$du = 2t\,dt$

$$= \int_2^0 -\sin u \, du = \left[\cos u\right]_2^0 = \cos 0 - \cos 2$$

$$= 1 - \cos 2.$$

4. (4 points) Find an explicit formula for the following recursively defined sequence:

$$a_n = (-2) \cdot a_{n-1} \text{ for } n \geq 2,\ a_1 = 6.$$

$$a_n = (-2)\, a_{n-1} = (-2)\cdot(-2)\, a_{n-2}$$

$$= \cdots$$

$$= \underbrace{(-2)(-2)\cdots(-2)}_{n-1\ 2\text{'s.}} \cdot a_1$$

$$= (-2)^{n-1} \cdot 6$$

5. (6 points) The "explain" problem.

Explain why $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges when $p > 1$, without using the Yardstick Theorem.

At least half credit will be given for showing convergence when $p = 3$.

Since $\frac{1}{x^p}$ is cts, positive, decreasing on $[1, \infty)$,

IT says series is equiconvergent w/ $\int_1^{\infty} \frac{1}{x^p}\,dx$.

Calculate:

$$\int_1^{\infty} \frac{1}{x^p}\,dx = \left[\frac{x^{-p+1}}{-p+1}\right]_1^{\infty} = \lim_{8 \to \infty} \frac{1}{8^{p-1}} - \frac{1}{-p+1}$$

$$= 0 + \frac{1}{p-1}$$

since $p > 1$.

Since integral converges, series also converges. ✓

6. (6 points each) For each of the following series, determine whether it is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+2n}$

<u>Solution 1: Telescope!</u>

$$\frac{1}{n^2+2n} = \frac{A}{n} + \frac{B}{n+2} \Rightarrow 1 = A(n+2) + Bn$$

$$\Rightarrow A + B = 0,$$
$$2A = 1$$
$$\Rightarrow A = 1/2, \quad B = -\tfrac{1}{2}.$$

Partial sum $s_N = \left(\frac{1/2}{1} - \frac{1/2}{3}\right) + \left(\frac{1/2}{2} - \frac{1/2}{4}\right) + \left(\frac{1/2}{3} - \frac{1/2}{5}\right) + \cdots$

$$\cdots + \left(\frac{1/2}{N-1} - \frac{1/2}{N+1}\right) + \left(\frac{1/2}{N} - \frac{1/2}{N+2}\right)$$

$$= 1/2 + 1/4 - \frac{1}{2(N+1)} - \frac{1}{2(N+2)}$$

$\longrightarrow 3/4$. So series converges to $3/4$.

<u>Solution 2: DCT</u>

$n^2+2n > n^2 \Rightarrow \frac{1}{n^2+2n} < \frac{1}{n^2}$, and since $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges (Yardstick)

$\sum_{n=1}^{\infty} \frac{1}{n^2+2}$ also converges.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3n^2+1}$

<u>Solution 1: IT</u>

Since $\frac{x}{3x^2+1}$ is cts, positive, decreasing on $[1,\infty)$, series is equiconvergent w/

$$\int_1^{\infty} \frac{x}{3x^2+1}\,dx = \int_4^{\infty} \frac{1}{6u}\,du = \left[\tfrac{1}{6}\ln u\right]_4^{\infty}$$

$u = 3x^2+1$
$du = 6x\,dx$

$$= \lim_{B\to\infty} \tfrac{1}{6}\ln B - \tfrac{1}{6}\ln 4$$

$$= \infty$$

Since integral diverges, series also diverges

<u>Solution 2: DCT</u>

Since $3n^2+1 \le 3n^2+n^2 = 4n^2$ for $n \ge 1$

Have $\frac{n}{3n^2+1} \ge \frac{n}{4n^2} = \frac{1}{4n}$ for $n \ge 1$.

Since $\sum_{n=1}^{\infty} \frac{1}{4n}$ [redacted] diverges (Yardstick Thm),

the given series also diverges.