MA 2733
Final Examination – December 10, 2014

Name _____

7 T/F, several long answer. 78 points.

General Instructions: Please answer the following, without use of calculators. You may refer to up to 4 3x5 cards or a single letter-sized sheet of paper, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ The vector $\langle 2, 4, -2\rangle$ is normal to the plane $3x + 6y - 3z = 4$.

   (b) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are orthogonal vectors, then $(\vec{\mathbf{v}} + \vec{\mathbf{u}}) \bullet (\vec{\mathbf{v}} + \vec{\mathbf{u}}) = \|\vec{\mathbf{v}}\|^2 + \|\vec{\mathbf{u}}\|^2$.

   (c) ____ The function $f(x) = \sqrt{x}$ has a power series representation centered at $a = 0$.

   (d) ____ The function $f(x) = \ln(1 + x)$ has a power series representation centered at $a = 0$.

   (e) ____ If $\vec{\mathbf{v}}$, $\vec{\mathbf{u}}$, and $\vec{\mathbf{w}}$ are vectors in $\mathbb{R}^3$ such that $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are orthogonal, and $\vec{\mathbf{u}}$ and $\vec{\mathbf{w}}$ are orthogonal, then $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are also orthogonal.

   (f) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ are vectors such that $\vec{\mathbf{v}} \bullet \vec{\mathbf{w}} = 1/2$, then the angle between $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is $\pi/3$.

   (g) ____ If $0 \leq d_n \leq \pi/n$ for all $n \geq 5$, then $\displaystyle\sum_{n=1}^{\infty} \frac{d_n}{n}$ converges.

2. Lines and planes

(a) (4 points) Find a plane which is parallel to the plane $x - 2y - z = 2$, and which passes through the point $(1, 1, 1)$.

(b) (4 points) Find a plane which is orthogonal to the line $\vec{\mathbf{r}}(t) = t\langle 2, 2, -2\rangle + \langle -1, -1, 0\rangle$, and which passes through the origin.

(c) (4 points) Find any point which has a distance of 2 from the plane $x-2y-z = 2$.

3. Series convergence

(a) (6 points) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{5^n}{(2n)!} x^n$.

(b) (6 points) Discuss convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2 \sin n \cdot \cos n}{n\sqrt{n}}$.

4. Power series representations

(a) (6 points) Find a power series representation for $\dfrac{\sin x^3}{x^2}$.

(b) (6 points) Find a series converging to $\displaystyle\int_0^2 \sin x^3 \, dx$.

5. Traces

(a) (2 points) Find the trace of the surface $z = x^2 + e^y$ in the xy plane.

(b) (4 points) Find the trace of the surface $z = x^2 + e^y$ in at least 3 planes that are parallel to the xz plane.

(c) (4 points) Find the trace of the surface $z = x^2 + e^y$ in at least 3 planes that are parallel to the yz plane.

6. The TNB frame and curvature

   (a) (5 points) Find the curvature $\kappa(t)$ for the curve $\vec{\mathbf{r}}(t) = \langle t^2, 2t, t^3 \rangle$.

   (b) (4 points) For the curve $\vec{\mathbf{r}}(t)$, calculate the unit tangent vector $\vec{\mathbf{T}}(t)$.

   (c) (4 points) For the curve $\vec{\mathbf{r}}(t) = \langle t^2, 2t, t^3 \rangle$, write $\vec{\mathbf{r}}''(t)$ in terms of the $\vec{\mathbf{T}}, \vec{\mathbf{N}}, \vec{\mathbf{B}}$ frame.

7. (5 points) Suppose that $y = f(x) = \sum_{n=0}^{\infty} a_n x^n$ is a solution to the differential equation $y'' - y' - y = 0$. Give a recursive formula for a_n that holds for any $n \geq 2$.