MA 2733

Examination 3 – November 19, 2014

Name ______________________________

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ______________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ If $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{w}}$ are vectors, then $(\vec{\mathbf{u}} \bullet \vec{\mathbf{v}}) \times \vec{\mathbf{w}}$ is orthogonal to $\vec{\mathbf{w}}$.

   (b) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are orthogonal vectors, then $\|\vec{\mathbf{v}} \times \vec{\mathbf{u}}\| = \|\vec{\mathbf{v}}\| \cdot \|\vec{\mathbf{u}}\|$.

   (c) ____ If $\vec{\mathbf{u}}$, $\vec{\mathbf{v}}$, and $\vec{\mathbf{w}}$ are vectors, and $\vec{\mathbf{w}}$ is orthogonal to $\vec{\mathbf{v}} \times \vec{\mathbf{u}}$, then either $\vec{\mathbf{w}} = \vec{\mathbf{v}} \times \vec{\mathbf{u}}$ or else $\vec{\mathbf{w}} = \vec{\mathbf{u}} \times \vec{\mathbf{v}}$.

   (d) ____ If $\vec{\mathbf{u}}$ and $\vec{\mathbf{v}}$ are vectors, then $\|\vec{\mathbf{u}} + \vec{\mathbf{v}}\| = \|\vec{\mathbf{u}}\| + \|\vec{\mathbf{v}}\|$.

   (e) ____ If $\vec{\mathbf{u}}$ is a vector, then $\dfrac{\vec{\mathbf{u}}}{\vec{\mathbf{u}} \bullet \vec{\mathbf{u}}}$ is a unit vector.

2. Lines and planes

(a) (4 points) Find a vector orthogonal to the plane $2x + 3y - 2z = 4$, and a point that lies on the plane.

(b) (4 points) Find 2 unit vectors that are parallel to the vector $\langle 3, -1, 4 \rangle$.

(c) (5 points) Do the points $(1, 2, 3)$, $(3, 1, 3)$, $(3, 2, 2)$, and $(5, 1, 2)$ lie on a common plane? If so, find the plane! If not, explain why not.

(d) (4 points) Give the vector equation of a line that passes through the point $(0, 1, 1)$ and that is parallel to the line given by the symmetric equations

$$x - 1 = \frac{y - 2}{2} = \frac{z - 3}{3}.$$

3. Vector functions and curves.

(a) (5 points) Find the unit tangent vector $\vec{\mathbf{T}}(t)$ for the curve $\vec{\mathbf{r}}(t) = \langle t, \frac{t^2}{2}, \frac{t^3}{3} \rangle$.

(b) (4 points) For the curve $\vec{\mathbf{r}}(t) = \langle 2\sin t, \cos t, -2t \rangle$, find $\vec{\mathbf{r}}'(t)$ and $\vec{\mathbf{r}}''(t)$.

(c) (4 points) For the curve $\vec{\mathbf{r}}(t) = \langle 2\sin t, \cos t, -2t \rangle$ from part (b), find the plane which contains the point $\vec{\mathbf{r}}(\pi)$ and is parallel to the vectors $\vec{\mathbf{r}}'(\pi)$ and $\vec{\mathbf{r}}''(\pi)$.

(d) (4 points) Give a vector function representation for the curve given by the trace in the xy-plane of the curve $x^2 + 2y^2 + z^2 = 4$.

4. (6 points) The "explain" problem.
Using only the definition of dot product and elementary geometry, show that if the angle between 3-dimensional vectors $\vec{\mathbf{v}}$ and $\vec{\mathbf{w}}$ is $\pi/2$, then $\vec{\mathbf{v}} \bullet \vec{\mathbf{w}} = 0$.

Note: the definition of the dot product is the formula you usually use to compute it, and in particular does not involve any trig functions.