MA 2733
Examination 2 – October 22, 2014

Name _____

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ If $\frac{1}{n^2} \leq a_n \leq \frac{1}{\sqrt{n}}$ for all $n \geq 3$, then $\sum_{n=1}^{\infty} a_n$ diverges.

   (b) ____ If $\frac{1}{n^3} \leq a_n \leq \frac{1}{n^{3/2}}$ for all $n \geq 3$, then $\sum_{n=1}^{\infty} a_n$ converges.

   (c) ____ If the power series $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = 4$, then it must converge at $x = -\pi$.

   (d) ____ The coefficient of x^{10} in $\sum_{m=1}^{\infty} m \cdot x^{2m}$ is 10.

   (e) ____ Every function that can be differentiated infinitely many times has a Taylor series.

2. (9 points) Discuss convergence of the series $\sum_{n=0}^{\infty} \frac{-\cos n}{2^n + 1}$. (Determine whether it is absolutely convergent, conditionally convergent, or divergent.)

3. Power series convergence: $\sum_{n=0}^{\infty} \frac{e^{-n}}{\sqrt{n} + 2} \cdot x^n$

   (a) (6 points) Find the radius of convergence of the above power series.

(b) (6 points) Find the interval of convergence of the above power series.

4. (6 points) The "explain" problem. Show that if $\sum_{n=0}^{\infty} a_n$ is absolutely convergent, then it is convergent.

5. Power series applications

(a) (5 points) Find a power series representation for $f(x) = e^{3x^2}$.

(b) (5 points) Using your answer from part (a), find a series representation for $\int_0^1 e^{3x^2}\,dx$.

(c) (3 points) Using your answer from part (a) and Taylor's Theorem, find $f^{(99)}(0)$ and $f^{(100)}(0)$ for $f(x) = e^{3x^2}$.