

MA 2733

Examination 1 – September 24, 2014

Name \_\_\_\_\_

5 T/F, several long answer. 50 points.

**General Instructions:** Please answer the following, without use of calculators.

You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section).

You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

**Mississippi State University Honor Code:** “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.”

Signature \_\_\_\_\_

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) \_\_\_\_\_  $\sum_{n=1}^{\infty} a_n = L$  means that  $\lim_{m \rightarrow \infty} \sum_{n=1}^m a_n = L$ .

(b) \_\_\_\_\_ Assuming that the series  $\sum_{n=1}^{\infty} \frac{1}{n^4 - \pi}$  converges, the integral  $\int_2^{\infty} \frac{1}{x^4 - \pi} dx$  must also converge by the Integral Test.

(c) \_\_\_\_\_ As  $n \rightarrow \infty$ , the sequence  $\frac{1}{\sqrt{n}}$  converges.

(d) \_\_\_\_\_ The series'  $\sum_{n=0}^{\infty} \frac{a_n}{2n+1}$  and  $\sum_{n=1}^{\infty} \frac{a_{n-1}}{2n+1}$  are the same.

(e) \_\_\_\_\_ If  $a_n$  is an increasing sequence that converges to 12, then  $a_n$  is bounded.

2. Parametric and polar equations

(a) (7 points) Find the tangent line to  $r = 2 \sin \theta$  at  $\theta = \frac{\pi}{3}$ .

(b) (7 points) Find the area that is inside the polar curve  $r = 2 \sin \theta$ , but outside the polar curve  $r = 1$ .

At least 3 points will be given for finding the area enclosed by  $r = 2 \sin \theta$ .

3. The “explain” problem.

(a) (2 points) State the arc-length formula for a parametric curve  $x = f(t)$ ,  $y = g(t)$ . Make sure to explain any additional unknowns (‘letters’ beyond  $x, y, t, f, g$ ) that you use.

(b) (5 points) Starting from the formula in (a), find the arc-length for a polar curve  $r = h(\theta)$  for  $\theta$  between  $c$  and  $d$ . (Explain why your formula works!)

4. Discuss convergence of the following series: determine whether each is convergent or divergent.

(a) (6 points)  $\sum_{n=0}^{\infty} \frac{n}{\sqrt{n^2 + 1}}$ .

(b) (7 points)  $\sum_{n=2}^{\infty} \frac{1}{n^2 - 1}$ .

(c) (6 points)  $\sum_{n=2}^{\infty} \frac{1}{n (\ln n)^{3/2}}$ .