

MA 2733

Final Examination – December 6, 2013

Name _____

7 T/F, several long answer. 62 points.

General Instructions: Please answer the following, without use of calculators.

You may refer to up to 4 3x5 cards or a single letter-sized sheet of paper, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.”

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) _____ The sequence a_n , recursively defined sequence by $a_0 = 3$, $a_n = \frac{1}{2}a_{n-1}$ for $n \geq 1$, is a convergent sequence.

(b) _____ $x^2 - 3x + 5$ can be considered to be a power series.

(c) _____ The planes given by equations $x - y + z = 2$ and $-x + y - z = 3$ are parallel.

(d) _____ Every function has a power series representation around $a = 0$.

(e) _____ $(\vec{v} + \vec{u}) \times (\vec{v} + \vec{u}) = \|\vec{v}\|^2 + \|\vec{u}\|^2$

(f) _____ If $\vec{r}(t)$ is a vector function with unit normal vector $\vec{T}(t)$, then

$$(\vec{T} + \vec{T}') \cdot (\vec{T} + \vec{T}') = \|\vec{T}'\|^2 + 1.$$

(g) _____ If $(\vec{a} \times \vec{b}) \cdot \vec{c} = 0$, then \vec{c} is orthogonal to both \vec{a} and \vec{b} .

2. (8 points) Find the curvature of the vector function $\vec{\mathbf{r}}(t) = \langle 2 \sin x + 1, 2 \cos x + 1, 4t - 2 \rangle$.

3. (6 points) Find the equation of a line orthogonal to the plane $x - y + z = 1$, and determine where your line intersects the plane.

4. (7 points) Working directly from the definition, find the Taylor series for e^{2x} .
Partial credit of at least 3 points will be given for instead finding the Taylor series via power series operations.

5. (9 points) Find a power series representation (around $a = 0$) for $\frac{1}{x^2+2x+2}$.

6. (11 points) Find the radius and interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{n}{n+3} x^n.$$

7. (7 points) Find the area inside the polar curve $r = 2 + \cos \theta$.