MA 2733
Final Examination – December 6, 2013

Name _____

7 T/F, several long answer. 62 points.

General Instructions: Please answer the following, without use of calculators. You may refer to up to 4 3x5 cards or a single letter-sized sheet of paper, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ The sequence $\dfrac{1}{n!}$ converges.

   (b) ____ $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}\sin(nx)$ can be considered to be a power series.

   (c) ____ The planes given by equations $x - y + z = 3$ and $-x + y + z = 2$ are parallel.

   (d) ____ Every function that is infinitely differentiable (on $(-\infty, \infty)$ has a Taylor series around $a = 0$.

   (e) ____ $(\vec{\mathbf{v}} + \vec{\mathbf{u}}) \bullet (\vec{\mathbf{v}} + \vec{\mathbf{u}}) = \|\vec{\mathbf{v}}\|^2 + \|\vec{\mathbf{u}}\|^2$

   (f) ____ The twisted cubic $\vec{\mathbf{r}}(t) = \langle t, t^2, t^3 \rangle$ does not have a parametrization according to arc length.

   (g) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are unit vectors, then $\|\vec{\mathbf{v}} \times \vec{\mathbf{u}}\| = 1$.

2. (8 points) Find the curvature of the vector function $\vec{\mathbf{r}}(t) = \langle 3\sin x - 1, 3\cos x - 1, 6t + 2\rangle$.

3. (7 points) Find the equation of a plane containing the line $\langle x, y, z\rangle = \langle 3t - 1, t, t\rangle$ and the point $(0, 1, 1)$.

4. (7 points) Find the Taylor series around $a = \frac{\pi}{2}$ for $f(x) = \sin x$.

5. (8 points) Recall that the *hyperbolic cosine function* is defined as $\cosh x = \dfrac{e^x + e^{-x}}{2}$. Find a power series representation (around $a = 0$) for $\cosh x$.

6. (11 points) Find the radius and interval of convergence for the power series $\sum_{n=0}^{\infty} \frac{2^n}{n^2+2} x^n$.

7. (7 points) Consider the parametric curve $\langle x, y \rangle = \left\langle 2t^2, e^{t^3} \right\rangle$. For what values of t does the tangent line to the curve have a positive slope?
At least 4 points for computing an appropriate derivative.