MA 2733

Examination 2 – October 23, 2013

Name ________________________________

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ________________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) _____ If (a, b, c) is a point on the plane $\alpha x + \beta y + \gamma z = 0$, then $\langle a, b, c\rangle$ is orthogonal to $\langle \alpha, \beta, \gamma\rangle$.

   (b) _____ The twisted cubic $\vec{\mathbf{r}}(t) = \langle t, t^2, t^3\rangle$ does not have a parametrization according to arc length.

   (c) _____ If $\vec{\mathbf{r}}(t)$ is any vector function, then $\|\vec{\mathbf{r}}\| = \sqrt{\vec{\mathbf{r}} \bullet \vec{\mathbf{r}}}$.

   (d) _____ Suppose f is a continuous function with $\lim_{x\to n} f(x) = 0$. If $a_n = f(n)$, then $\lim_{n\to\infty} a_n = 0$.

   (e) _____ If a_n is a bounded sequence, then $\lim_{n\to\infty} a_n$ converges.

2. (10 points) Find the curvature of the vector function $\vec{\mathbf{r}}(t) = \left\langle t, \frac{t^2}{2}, \frac{t^3}{3} \right\rangle$ (as a function of t).

3. Sequences

   (a) (3 points) Find $\lim_{n\to\infty} \frac{1}{n}$. (Make sure to briefly explain your answer!)

(b) (8 points) Find an explicit formula for the sequence a_n which is given recursively by $a_n = 2n \cdot a_{n-1}$ for $n > 1$, with $a_1 = 5$.

4. Planes and surfaces

(a) (7 points) On the same axis, draw the traces of the surface $z = x \sin y$ in the planes $x = 0, 1,$ and 2.

(b) (5 points) Find 2 unit vectors parallel to the plane $2x - y - z = 2$.
Partial credit will be given for finding several distinct points on the plane.

5. (7 points) The "explain" problem.
Let $\vec{\mathbf{r}}(t)$ be a smooth vector function such that $\vec{\mathbf{r}}'$ is also smooth. Explain why

$$\vec{\mathbf{r}}''(t) = \nu\vec{\mathbf{T}} + \nu^2\kappa\vec{\mathbf{N}}.$$

At least 1 point will be given for stating clearly what ν and κ represent in this equation.