

MA 2733

Final Exam – December 7, 2012

Name \_\_\_\_\_

8 T/F, 4 long answer. 86 points.

**General Instructions:** Please answer the following, without use of calculators.

You may refer to a single sheet of paper (up to 8.5 x 11), but no other notes.

Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

**Mississippi State University Honor Code:** “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.”

Signature \_\_\_\_\_

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) \_\_\_\_\_ If  $\mathbf{v} \cdot \mathbf{w} = 0$ , then  $\mathbf{v} \times \mathbf{w} = 0$ .

(b) \_\_\_\_\_ If  $\mathbf{v} \times \mathbf{w} = 0$ , then  $\mathbf{v} \cdot \mathbf{w} = 0$ .

(c) \_\_\_\_\_ Any curve  $y = f(x)$  can also be represented as a parametric equation.

(d) \_\_\_\_\_ If  $\lim_{n \rightarrow \infty} c_n = 0$ , then the series  $\sum_{n=1}^{\infty} c_n$  converges.

(e) \_\_\_\_\_ If  $\mathbf{r}$  is any vector function, then  $\mathbf{r}' \cdot \mathbf{r}'' = 0$ .

(f) \_\_\_\_\_ The series  $\sum_{n=0}^{\infty} (-1)^n \cdot \sqrt{n}$  converges.

(g) \_\_\_\_\_ The sequence  $\frac{1}{\sqrt{n}}$  converges.

(h) \_\_\_\_\_ If  $\lim_{n \rightarrow \infty} (c_{n+1}/c_n) = e^{-1}$ , then the series  $\sum_{n=1}^{\infty} c_n$  converges.

2. (8 points) Find the radius and interval of convergence of the power series  $\sum_{n=0}^{\infty} \frac{3^n}{n - \sqrt{2}} \cdot x^n$ .

3. Vector equations

(a) (6 points) Find a vector that is orthogonal to  $\langle 2, -1, 3 \rangle$ .

(b) (5 points) Find the equation of a plane containing the lines  $\mathbf{r}(t) = \langle 1, 1, 1 \rangle t$  and  $\mathbf{s}(t) = \langle 0, 1, 1 \rangle t$ .

(c) (7 points) Graph the traces of the surface  $z = x^2 + \frac{y^2}{4}$  in the planes  $x = 1$ ,  $x = 2$ ; and in the planes  $y = 1$ ,  $y = 2$ .  
For full credit: label each axis and indicate which trace is which!

4. Calculus in 2 and 3 dimensions

(a) (8 points) Find the  $\mathbf{T}, \mathbf{N}, \mathbf{B}$  basis vectors (as functions of  $t$ ) for the vector function  $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$ .

(b) (7 points) Show that the curvature  $\kappa$  of the straight line  $\mathbf{r}(t) = \langle t, 2t, 3t \rangle$  is 0 for all  $t$ .

(c) (8 points) Find the length of the curve  $\mathbf{r}(t) = \langle e^{-t} \sin t, e^{-t} \cos t \rangle$ ,  $0 \leq t \leq \pi$ .

5. Power series representations

(a) (8 points) Find a power series representation of  $\int \cos x^2 dx$ .

(b) (6 points) For the function  $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$ , find  $f(0)$ ,  $f^{(7)}(0)$ , and  $f^{(8)}(0)$ .

(c) (7 points) Using the definition, show that the Taylor series for  $e^x$  is  $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$ .