MA 2733

Examination 3 – November 28, 2012

Name _____

5 T/F, 4 long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) _____ If $\lim_{n\to\infty} b_n = 0$, then the series $\sum_{n=1}^{\infty} b_n$ converges.

   (b) _____ If the series $\sum_{n=1}^{\infty} b_n$ converges, then the series $\sum_{n=1}^{\infty} |b_n|$ converges.

   (c) _____ If the series $\sum_{n=1}^{\infty} |b_n|$ converges, then the series $\sum_{n=1}^{\infty} b_n \cdot \sin n$ converges.

   (d) _____ If the power series $\sum_{n=3}^{\infty} c_n x^n$ converges at $x = 3$, then it converges at $x = e$.

   (e) _____ If the power series $\sum_{n=0}^{\infty} c_n x^n$ converges at $x = 3$, then it converges at $x = -3$.

2. Discuss convergence of the following series: determine whether each is absolutely convergent, conditionally convergent, or divergent.

   (a) (6 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n \cdot n^2 \cdot 3^n}{4^n}$.

   (b) (7 points) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{\sqrt{n}+2}$.

3. (8 points) Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(n^2+1)\cdot 2^n}$.

4. Power series representations

    (a) (6 points) Find a power series representation (around $a = 0$) for $\ln(x - 2)$.
        At least 3 points will be given for a power series representation of $\dfrac{1}{x - 2}$.

    (b) (7 points) Find a power series representation (around $a = 0$) for $\dfrac{d^3}{dx^3}\left(\dfrac{1}{1 - x^2}\right)$.

5. (6 points) The "explain" problem.

    Explain why $\displaystyle\sum_{n=0}^{\infty} r^n = \dfrac{1}{1 - r}$ when $|r| < 1$.