MA 2733

Final Exam – December 11, 2012

Name ______________________________

8 T/F, 4 long answer. 86 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a single sheet of paper (up to 8.5 x 11), but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ______________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ If $\mathbf{v} \times \mathbf{w} = \mathbf{0}$, then $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

   (b) ____ If $\mathbf{v} \cdot \mathbf{w} = 0$, then $\mathbf{v}$ and $\mathbf{w}$ are orthogonal.

   (c) ____ The sequence $\frac{1}{\sqrt{n}}$ converges.

   (d) ____ The series $\sum_{n=0}^{\infty} (-1)^n \cdot \frac{n}{n+1}$ converges.

   (e) ____ If $\mathbf{r}$ is any vector function, then $\mathbf{r}' \cdot \mathbf{r}'' = 0$.

   (f) ____ If $\lim_{n\to\infty} (c_{n+1}/c_n) = e$, then the series $\sum_{n=1}^{\infty} c_n$ converges.

   (g) ____ Any polar curve $r = f(\theta)$ can also be represented as a parametric equation.

   (h) ____ If $\lim_{n\to\infty} c_n = 0$, then the series $\sum_{n=1}^{\infty} c_n$ converges.

2. (8 points) Find the radius and interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{4^n}{n-\sqrt{2}} \cdot x^n$.

3. Vector equations

(a) (6 points) Give a vector equation for the line $y = 3x - 2$.

(b) (5 points) Find the equation of a plane containing the lines $\mathbf{r}(t) = \langle 1, 1, 0 \rangle\, t + \langle 1, 0, 1 \rangle$ and $\mathbf{s}(t) = \langle 0, 1, 1 \rangle\, t$.

(c) (7 points) Graph the traces of the surface $z = x^2 + \dfrac{y}{2}$ in the planes $x = 1$, $x = 2$; and in the planes $y = 1$, $y = 2$.
For full credit: label each axis and indicate which trace is which!

4. Calculus in 2 and 3 dimensions

(a) (8 points) Find the $\mathbf{T}, \mathbf{N}, \mathbf{B}$ basis vectors (as functions of t) for the vector function $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$.

(b) (7 points) Show that if $\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$ has curvature function $\kappa(t)$, then $\mathbf{s}(t) = \langle x(t) + 2, y(t) - 6, z(t) + 2 \rangle$ has the same curvature function.

(c) (8 points) Find the length of the curve $\mathbf{r}(t) = \langle e^{-t} \sin t, e^{-t} \cos t \rangle$, $0 \leq t \leq \pi$.

5. Power series representations

(a) (8 points) Find a power series representation of $\int xe^{x^2}\,dx$.

(b) (6 points) For the function $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$,find $f(0)$, $f^{(7)}(0)$, and $f^{(8)}(0)$.

(c) (7 points) Using the definition, show that the Taylor series for $\cos x$ is $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} x^k$.