

MA 2733

Final Exam – December 7, 2012

Name _____

8 T/F, 4 long answer. 86 points.

General Instructions: Please answer the following, without use of calculators.

You may refer to a single sheet of paper (up to 8.5 x 11), but no other notes.

Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: “As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do.”

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

(a) _____ If $\mathbf{v} \times \mathbf{w} = 0$, then $\mathbf{v} \cdot \mathbf{w} = 0$.

(b) _____ If $\mathbf{v} \cdot \mathbf{w} = 0$, then $\mathbf{v} \times \mathbf{w} = 0$.

(c) _____ If $\lim_{n \rightarrow \infty} (c_{n+1}/c_n) = e^{-1}$, then the series $\sum_{n=1}^{\infty} c_n$ converges.

(d) _____ The sequence $\frac{1}{\sqrt{n}}$ converges.

(e) _____ The series $\sum_{n=0}^{\infty} (-1)^n \cdot \sqrt{n}$ converges.

(f) _____ Any curve $y = f(x)$ can also be represented as a parametric equation.

(g) _____ If \mathbf{r} is any vector function, then $\mathbf{r}' \cdot \mathbf{r}'' = 0$.

(h) _____ If $\lim_{n \rightarrow \infty} c_n = 0$, then the series $\sum_{n=1}^{\infty} c_n$ converges.

2. (8 points) Find the radius and interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{3^n}{n - \sqrt{2}} \cdot x^n$.

3. Vector equations

(a) (6 points) Find a vector that is orthogonal to $\langle 2, -1, 3 \rangle$.

(b) (5 points) Find the equation of a plane containing the lines $\mathbf{r}(t) = \langle 1, 1, 1 \rangle t$ and $\mathbf{s}(t) = \langle 0, 1, 1 \rangle t$.

(c) (7 points) Graph the traces of the surface $z = x^2 + \frac{y^2}{4}$ in the planes $x = 1$, $x = 2$; and in the planes $y = 1$, $y = 2$.
For full credit: label each axis and indicate which trace is which!

4. Calculus in 2 and 3 dimensions

(a) (8 points) Find the $\mathbf{T}, \mathbf{N}, \mathbf{B}$ basis vectors (as functions of t) for the vector function $\mathbf{r}(t) = \langle \cos t, \sin t, 3t \rangle$.

(b) (7 points) Show that the curvature κ of the straight line $\mathbf{r}(t) = \langle t, 2t, 3t \rangle$ is 0 for all t .

(c) (8 points) Find the length of the curve $\mathbf{r}(t) = \langle e^{-t} \sin t, e^{-t} \cos t \rangle$, $0 \leq t \leq \pi$.

5. Power series representations

(a) (8 points) Find a power series representation of $\int \cos x^2 dx$.

(b) (6 points) For the function $f(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2}$, find $f(0)$, $f^{(7)}(0)$, and $f^{(8)}(0)$.

(c) (7 points) Using the definition, show that the Taylor series for e^x is $\sum_{n=0}^{\infty} \frac{1}{n!} x^n$.