MA 2733

Examination 2 – October 23, 2013

Name ________________________________

5 T/F, several long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature ________________________________

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ The curvature of the line $\vec{\mathbf{r}}(t) = \langle t, t, t \rangle$ is not defined.

   (b) ____ For the vector function $\vec{\mathbf{r}}(t) = \langle t, t^2, 2t^3 \rangle$, the unit normal vector $\vec{\mathbf{N}}$ is orthogonal to $\vec{\mathbf{N}}'$.

   (c) ____ If a_n is an increasing sequence, then $\lim_{n\to\infty} a_n$ converges.

   (d) ____ Suppose f is a continuous function with $\lim_{x\to\infty} f(x) = 0$. If $a_n = f(n)$, then $\lim_{n\to\infty} a_n = 0$.

   (e) ____ If $\vec{\mathbf{r}}(t)$ is a vector function with unit normal vector $\vec{\mathbf{T}}(t)$, then

$$\left(\vec{\mathbf{T}} + \vec{\mathbf{T}}'\right) \bullet \left(\vec{\mathbf{T}} + \vec{\mathbf{T}}'\right) = \left\|\vec{\mathbf{T}}\right\|^2 + \left\|\vec{\mathbf{T}}'\right\|^2.$$

2. (11 points) Write $\vec{\mathbf{r}}''(t)$ in the $\vec{\mathbf{T}}, \vec{\mathbf{N}}, \vec{\mathbf{B}}$ frame for the vector function $\vec{\mathbf{r}}(t) = \langle 4\cos t, 4\sin t, 3t\rangle$. (You need not calculate $\vec{\mathbf{T}}, \vec{\mathbf{N}}$, or $\vec{\mathbf{B}}$.)

3. (9 points) Find $\lim_{n\to\infty} \dfrac{2^{\sin(\frac{\pi}{2}n)}}{n^2}$. (Make sure you explain how you found your answer!)

4. Planes and surfaces

(a) (7 points) On the same axis, draw the traces of the surface $z = xy$ in the planes $x = 0, 1,$ and 2.

(b) (2 points) Consider the line parallel to the vector $\langle 2, 1, 2 \rangle$ and passing through $(1, 0, 0)$. Find a point other than $(1, 0, 0)$ on this line.

(c) (3 points) Find 3 distinct points on the plane $x - y - 2z = 1$.

5. The "explain" problem.

(a) (4 points) State and verify the "sum rule" for $\frac{d}{dt}[\vec{\mathbf{v}}(t) + \vec{\mathbf{u}}(t)]$.

(b) (4 points) State and verify the "dot product rule" for $\frac{d}{dt}[\vec{\mathbf{v}}(t) \bullet \vec{\mathbf{u}}(t)]$.