MA 2733
Examination 1 – September 25, 2013

Name _____

5 T/F, 3 long answer. 50 points.

General Instructions: Please answer the following, without use of calculators. You may refer to a 3x5 card, but no other notes. Correct answers without correct supporting work may not receive full credit (excluding the True/False section). You may use the back of each page for additional answer space (please clearly indicate if you have done so), or scratch work.

Mississippi State University Honor Code: "As a Mississippi State University student I will conduct myself with honor and integrity at all times. I will not lie, cheat, or steal, nor will I accept the actions of those who do."

Signature _____

1. True/False. Enter T or F in each blank. A correct answer is worth 2 points, a blank space is worth 0 points, and a wrong answer is worth -2 points. (Your total on this problem will be rounded up to zero if necessary.)

   (a) ____ If $\vec{\mathbf{v}}$ and $\vec{\mathbf{u}}$ are orthogonal vectors, then $\|\vec{\mathbf{v}} \times \vec{\mathbf{u}}\| = 1$.

   (b) ____ The curve described by the symmetric equation

   $$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

   can also be described with a parametric equation.

   (c) ____ The Right-Hand Rule shows that $\vec{\mathbf{v}} \times \vec{\mathbf{w}}$ is orthogonal to $\vec{\mathbf{v}}$.

   (d) ____ If $(\vec{\mathbf{a}} \times \vec{\mathbf{b}}) \bullet \vec{\mathbf{c}} = 0$, then $\vec{\mathbf{c}}$ is orthogonal to both $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$.

   (e) ____ If $\vec{\mathbf{a}} \bullet \vec{\mathbf{a}} = 1$, then $\vec{\mathbf{a}}$ is a unit vector.

2. Parametric and polar equations

(a) (4 points) Set up an integral for the arc length of the polar curve given by $r = 2\theta \sin\theta$ for θ on the interval $[0, \pi/2]$. (You need not evaluate the integral.)

(b) (7 points) Find the area inside the polar curve $r = \sqrt{\theta \sin\theta}$ for θ between 0 and π.

(c) (3 points) In 1-2 sentences, explain under what conditions it is possible to write a parametric curve as a function $y = f(x)$.

(d) (7 points) By calculating an appropriate 2nd derivative, show that the cycloid described by $x = 3t - 3\sin t$, $y = 3 - 3\cos t$ is concave down except for at the points where t is a multiple of 2π.
At least 3 points will be given for correctly calculating an appropriate 1st derivative.

3. Vectors and straight-line geometry

(a) (5 points) Give the vector equation of a line which passes through the points $(1, 2, 3)$ and $(7, -3, 4)$.

(b) (4 points) Find the normal vector to and a point on the plane given by equation $x - y - z = 1$.

(c) (4 points) Find a unit vector $\vec{\mathbf{v}}$ which is orthogonal to $\langle 3, 1 \rangle$, and a unit vector $\vec{\mathbf{w}}$ which is parallel to $\langle 3, 1 \rangle$.

4. (6 points) The "explain why"/proof problem
Explain why $\vec{\mathbf{a}} \times \vec{\mathbf{b}}$ is orthogonal to $\vec{\mathbf{a}}$. (You may use the lemma from class if you find it helpful.)