MA 2733

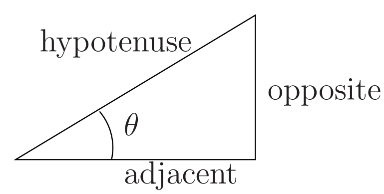
Trigonometry Overview – September 5, 2012

1. Definition

   (a) Ratios in right triangles:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

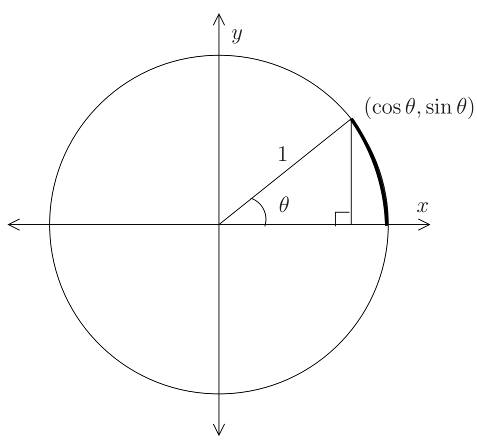
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$



Special case: When hypotenuse = 1, we have $\sin\theta$ = opposite and $\cos\theta$ = adjacent.)

A problem with this definition: What is $\sin\frac{3\pi}{2} = \sin 270°$?

   (b) Points on the circle with radius 1, which we call the *unit circle*: (Place a triangle)



Note: If $\theta = \frac{\pi}{4}$, then the length of the shaded part of the unit circle is $\frac{\pi}{4}$. (This is where radians come from!)

2. Useful values.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{2}$	$\frac{7\pi}{6}$
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1		
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0		

Use the symmetry in the circle above to fill in the blanks.

3. Identities

   (a) Pythagorean Theorem:

   $$\sin^2\theta + \cos^2\theta = 1.$$

   (95% of the time, this is what you'll use.)

   (b) Sums:

   $$\begin{aligned} \sin(x+y) &= \sin x \cos y + \cos x \sin y \\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \end{aligned}$$

4. Derivatives and integrals and limits

   $$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

   $$\begin{aligned} \frac{d}{dx}\sin x &= \cos x \quad , \qquad \frac{d}{dx}\cos x = -\sin x \\ \int \sin x &= -\cos x + C \quad , \qquad \int \cos x = \sin x + C \end{aligned}$$

5. Other trig functions:

   $$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \csc x = \frac{1}{\sin x}, \quad \cot x = \frac{1}{\tan x}$$

   For many problems, you should start by expressing any other trig functions in terms of sin and cos.

6. Graphs. (Note that here, as always, sin and cos are in radians.)